K-Shell Decomposition of AS Level Multigraphs

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***Abstract*—The Internet is one of the immense human-engineered systems and understanding of the topology can be helpful for network engineers and researchers. Categorization of Autonomous Systems (ASes) plays an essential role in understanding the structure and evolution of the Internet. However, the traditional categorization exhibits variation in different studies, contains ambiguity, involves subjectiveness, and sometimes does not match the reality. A better approach to classify ASes is defining the AS level topology maps as graphs and taking advantage of the graph properties through k-shell decomposition. However, the proposed solutions neither capture the parallel connections nor incorporate the varying business relations among the ASes. Abstracting ASes without any internal structure is an oversimplification since the ASes in the Internet span over various geographic regions and often cover the same regions in part or whole. In this work, we introduce k-shell decomposition on AS level multigraphs and comparison with AS level graphs. The decomposition is based on pruning the graphs according to the nodes' connectivity pattern to generate a layered structure of the Internet. In our experiments, we analyze the structure of the shells and the connectivity structure of the Internet. Additionally, we compare top-20 ASes to understand the central core of the Internet. Our comparative results help us to understand the structure of the Internet better.**

***Index Terms*—Complex Network, AS graphs, AS multigraphs, Internet modeling, Internet topology, k-core**

I. Introduction

The Internet is a large-scale network of networks which is formed by tens of thousands of autonomous networks. A group of networks managed by one or more operators under a well-defined routing policy is called an Autonomous System (AS) in the Internet. ASes are connected in different forms, i.e., customer-to-provider, peer-to-peer, and sibling-to-sibling, to enable global Internet communication [2, 4]. A special type of AS is called Internet Service Provider (ISP), which is a business entity providing Internet access service to individual users and small businesses while getting the same service from one or more upstream ISPs. At the core of the Internet, a small number of ISPs peer with each other to attain the global communication infrastructure.

An IP packet in the Internet passes through several routers until it reaches the destination. The routers handling the packet on the way typically belong to different ASes. The ASes forming the communication infrastructure of the Internet cooperate with each other to carry traffic from one host to another. Besides the collaboration, these ASes also compete with each other to increase their market share. While the AS accommodating the originating host might be a small (e.g., campus or building size), the ASes which transit the packet on the way might span over larger regions such as multiple states (e.g., Cox in the US) or countries (e.g., Cogent Communications all around the world).

Traditionally, the ASes in the Internet are categorized as tier-1, tier-2, tier-3, and stub ASes [13]. Tier-1 ASes are very large scale ISPs which are provider-free ASes that peer with every other provider-free AS to ensure reachability to all destinations without purchasing IP transit. Tier-2 ASes are large scale ISPs that peers with some networks but still purchases transit from some upstream ISPs to reach at least some portion of the Internet. Tier-3 ASes are small scale ISPs that solely purchases transit from upstream ISPs to participate in the Internet. Stub ASes are virtually located at the edge of the Internet, which have a single provider and do not carry any transit traffic from other ASes.

The traditional taxonomy of the ASes in the Internet has significant problems including several different definitions in various works, subjectiveness, and ambiguity [5, 12]. In some works, tier-1 ASes are defined as ISPs that serve their coverage area only through settlement-free business relations. Another definition requires those ISPs to participate in the largest settlement-free clique to be called tier-1 ASes. The ambiguity in the definitions of tier-2 and tier-3 ASes causes additional challenges to classify the intermediate transit-providing ASes because one AS may appear to be in different tiers based on its relationship with various other providers. Additionally, a stub AS is often defined as an AS with a single upstream provider without any customers. However, an AS in the Internet is required to have at least two upstream providers in order to receive a unique AS Number [22]. In fact, unadvertised backup links cause the stub ASes to look like single-homed ASes in the Internet.

A better approach to classify ASes is defining the AS level topology maps as graphs and taking advantage of the graph properties using a complex networks perspective. Typically, AS level Internet topology maps abstract the topology as a graph $G = (V, E)$ where the vertex set, V, corresponds to the ASes and the edge set, E, represents the logical relations between the ASes. Researchers have been suggested k-shell (also known as k-core) decomposition of AS level graphs to classify the ASes [15, 20]. The decomposition starts by removing all nodes with only one link and assigns them to the first shell. Next, the method again removes all nodes with degree two or less from the remaining graph to create

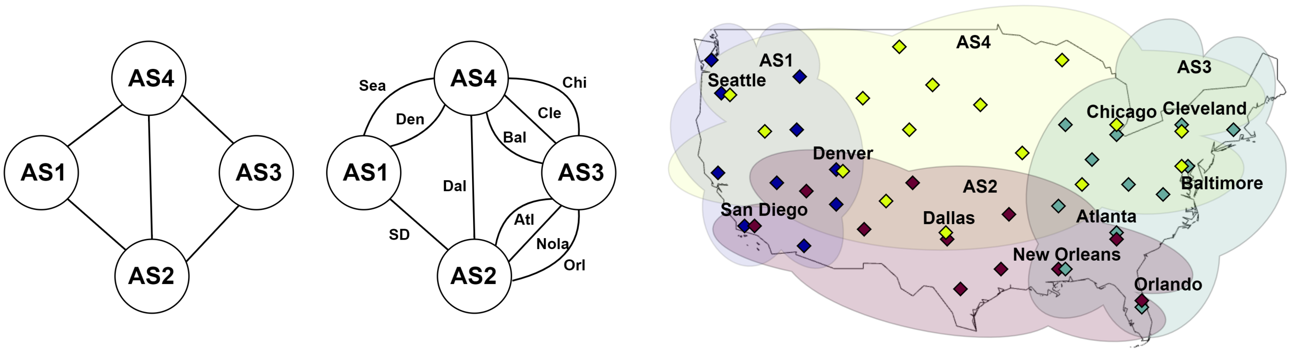


Fig. 1: Simple AS level graph and multigraph derived from an example topology map

the second shell. The method recursively applies the same operation until all nodes in the graph have been assigned to a shell.

The k-shell decomposition at AS level works well to classify ASes by using their relations with other ASes. However, the method ignores an important aspect of the real-world AS structures. Abstracting ASes without any internal structure is an oversimplification since the ASes in the Internet span over various geographic regions and often cover the same regions in part or whole [1, 5]. Moreover, they physically connect to each other at multiple colocation centers or Internet eXchange Points (IXPs) to exchange traffic and routing information. To illustrate, Figure 1 shows four ISPs, AS1, AS2, AS3, and AS4, providing Internet access service in the US. AS level Internet topology graph abstract the topology by using logical links where the nodes present ASes and the links present the business relations between ASes. Since the abstraction is binary, e.g., two ASes have a connection or not, it is inadequate to analyze the resilience and robustness of the Internet. In case a link failure occurs between two ASes in a certain location, AS level graphs are unable to show it because of the unavailability of location information. Therefore, if we analyze the link failure to assess network robustness in AS level graphs, we may not get the correct results for the Internet. ASes can still exchange their packets over different locations even if one of the locations has a link failure. As an example, Figure 1 shows that AS2 and AS3 have connections in Atlanta, New Orleans, and Orlando. AS level graphs show only one logical connection between AS2 and AS3. A proper abstraction of the Internet topology would be a multigraph $G = (V, E, f)$ where the vertex set, V, corresponds to the ASes, the edge multiset, E, represents the cross-connections between the ASes and $f : E \rightarrow \{(v_i, v_j) : v_i, v_j \in V, v_i \neq v_j\}$ is a function returning the endpoints of the edges to support parallel edges between two ASes [1]. Deriving the multigraph requires constructing a topology map that includes ASes, the cross-connections between the ASes, and abstraction for the endpoints of the connections.

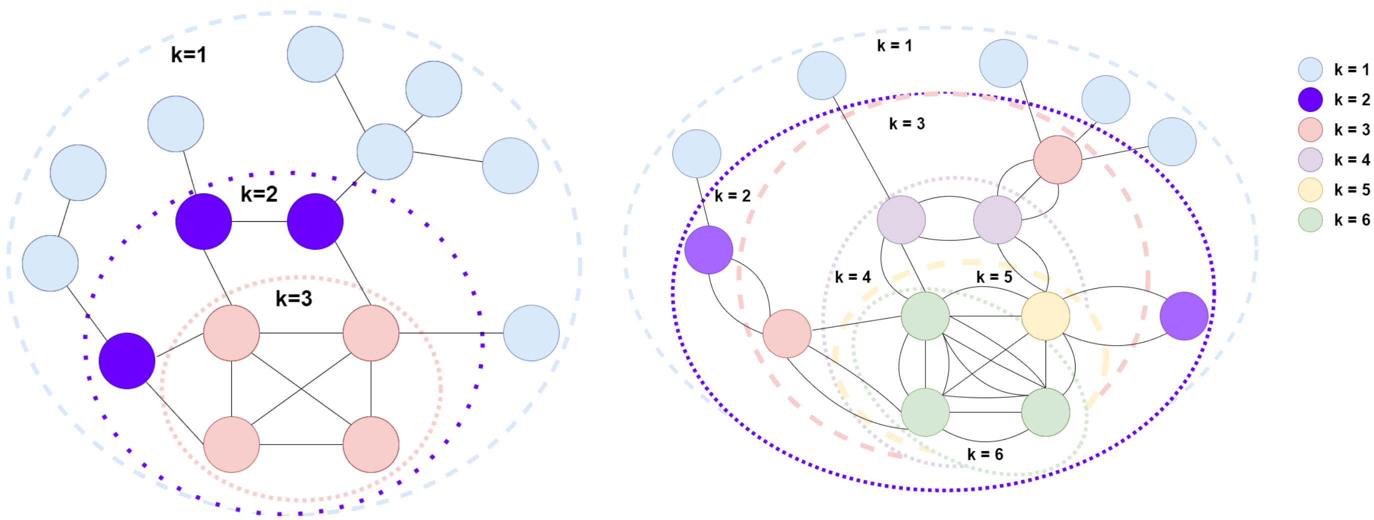
In this work, we use Cross-AS (X-AS) topology maps that we introduced in our previous work [1]. X-AS Internet topology maps capture both ASes and the parallel cross-AS connections observed at the network layer in the Internet. X-AS Internet topology maps allow us to abstract the AS level topology of the Internet as a multigraph rather than a simplified graph. We define k-shell decomposition on multigraphs to cluster ASes based on their parallel connections rather than the logical relations among them. Comparisons of k-shell decomposition between the multigraphs and graphs allow us to study the impact of parallel connections on AS clustering in the Internet.

The rest of the paper is organized as follows. In Section II, we present the related work. We introduce the details of our approach in Section III. Section IV demonstrates our experimental results. Finally, we conclude the paper in Section V.

II. Related Works

Ranking, classifying, and clustering the Autonomous Systems in the Internet is crucially vital for resilience and robustness analysis of the Internet, assessing the performance of protocols and routing algorithms, and improving the business relations between ISPs. Attack target detection is another significant motivation since attacking all ASes in the Internet is typically beyond the capability of cybercriminals. Therefore, targeting a small number of ASes, which results in the highest impact, is the best strategy for attackers. Similarly, network practitioners need to identify and secure those critical ASes to mitigate the impact of the attacks [6, 9]. Different measures have been introduced in the literature to rank, classify, and cluster the ASes in the Internet for various purposes.

In complex systems analysis, there are several graph measures to characterize graphs and assess the importance of nodes. Some of these metrics are degree, betweenness, eigenvector, and alpha centralities [10, 12]. Typically, the higher value in the centrality indicates the criticallity of a node. However, the concept of importance is versatile and depends on the application domain.



(a) AS Level Graphs

(b) AS Level Multigraphs

Fig. 2: Illustration of the Layered Structure of AS Level Graph and Multigraphs

Researchers used k-shell decomposition [16] in many areas, including social networks, visualization, computational biology, and Internet network analysis. In social networks, it has been used to analyze the influential spreaders [3]. In computational biology, researchers analyze the layer structure of the protein interaction network [7, 8]. Visualization for large-scale sparse network graphs also uses k-shell decomposition to layout several topological properties [11].

Hamelin et al. [20] apply k-shell decomposition to analyze the structure and self similarities of the Internet at the AS level. Carmi et al. [15] separate the network into three subcomponents; nucleus, fractal subcomponent, and dendrite-like structures. Nucleus core is defined as the main core where the ASes are well connected. Fractal subcomponent is a larger set that can connect each the majority of the Internet without congesting the nucleus ASes. Finally, the dendrite-like structures are isolated nodes that get service from nucleus ASes. Gregori et al. [21] study the k-clique communities in the Internet where each k-clique can be reached from one or series of adjacent k-clique ASes.

In this work, we use AS level multigraphs in addition to AS level graphs. Multigraphs allow us to analyze the parallel connections between ASes, which is a more realistic and adequate representation of the current Internet.

III. Methodology

In this section, we present our methodology to classify ASes by using k-shell decomposition. First, we describe the AS level graph and AS level multigraph generation process. Second, we give details about the k-shell decomposition method for both graphs and multigraphs.

A. *Topology Generation*

In this subsection, we present the AS level topology graph and multigraph generation process by using X-AS topology maps that we introduced in our previous work [1].

X-AS topology maps infer the AS level internet topology by capturing both ASes and cross-connections between the ASes. The maps allow us to abstract the AS level topology of the Internet as a multigraph rather than a graph. X-AS level topology maps use a set of techniques that exploit multiple data sources including, traceroute data, BGP advertisements, geolocation databases, and DNS datasets. We use traceroute datasets and IP address to AS mapping tools to extract IP addresses that appear in path traces where the paths switch from one AS to another. Then, we apply a set of techniques based on DNS names, geolocation databases, BGP advertisements, and traceroute datasets to accurately cluster IP addresses into their geolocations. Lastly, we exploit traceroute and BGP datasets to discover the cross-connections between the X-BI nodes. The final X-AS map, $X = (N, C)$, consists of a set of X-BI nodes, N, and a multiset of X-BI connections, C.

The final map can infer the simple AS level graphs and more informative AS level multigraphs. We define AS level graph G = (V; E) where the vertex set, V, corresponds to the ASes and the edge set, E, represents the logical relations between the ASes. Also we define AS level multigraph $G = (V, E, f)$ where the vertex set, V, corresponds to the ASes, the edge multiset, E, represents the cross connections between the ASes and $f : E \to \{(v_i, v_j) : v_i, v_j \in V, v_i \neq v_j\}$ is a function returning the endpoints of the edges to support parallel edges between two ASes.

B. *k-shell Decomposition*

In this subsection, we give details about our modified k-shell decomposition technique. For comparative purposes, we examine the problem in two steps: AS graphs and AS multigraphs.

The k-shell decomposition in complex network is a method to group nodes based on their interaction with the other nodes. The decomposition starts by removing all nodes having only one link and assigns them to the first shell. Next, the

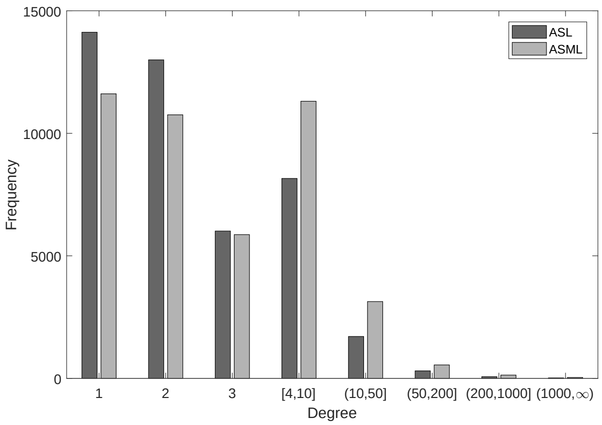


Fig. 3: AS Level Graph and Multigraph Degree Distribution

method removes all nodes with degree two or less from the remaining graph to create the second shell. The method recursively applies the same operation until all nodes in the graph have been assigned to a shell. More formally, the k-shell decomposition for AS level graphs can be defined as follow [20]:

Let be G = (V, E) an AS level graph where the vertex set, V, corresponds to the ASes and the edge set, E, represents the logical relations between the ASes.

k-core graph: A sub-graph $G' = (V', E|V')$ induced by the set $V' \subseteq V$ is a k-core if and only if the degree of every node in G' is greater or equal than k.

k-shell: A vertex v has shell index k if it belongs to the *k-core* but not to *(k + 1)-core*.

k-shell subgraph: A k-shell S_k is composed by all the vertices with the shell index k. The k-core is thus the union of all shells S_c with $c \geq k$.

$\boldsymbol{k_{max}}$**:** The maximum k value such that S_k is not empty.

In addition to the traditional graph definition of the k-shell decomposition, we define the multigraph version.

Let be $G = (V, E, f)$ where the vertex set, V, corresponds to the ASes, the edge multiset, E, represents the cross connections between the ASes and $f : E \rightarrow \{(v_i, v_j) : v_i, v_j \in V, v_i \neq v_j\}$ is a function returning the endpoints of the edges to support parallel edges between two ASes.

k-core multigraph: A sub-multigraph $G' = (V', E|V', f)$ induced by the set $V' \subseteq V$ is a k-core if and only if the degree of every node in G' is greater or equal than k.

Figure 2 presents an example of the k-shell decomposition of AS level graphs and multigraphs. The color code shows the values of each shell. Figure 2a shows the AS level graphs where 4 ASes are in the core with a k value of 3. On the other hand, if we check the same topology's multigraph version in Figure 2b, we observe that only 3 of the ASes are in the main core with a k value 6. We modified the algorithm proposed by Batagelj and Zaversnik [17]. The algorithm has time complexity $O(|V| + |E|)$.

TABLE I: Summary Statistics for AS Degree Distribution

$Type$	Q_0	Q_1	Q_2	Q_3	Q_4	$Mean$	$StdDev$
ASL	1	1	2	3	6365	5.33	59.29
ASML	1	1	2	5	26150	9.96	191.79

TABLE II: Top-10 ASes Degree Comparison (ASL Sorted)

ASN	Company	ASL	ASML	$\frac{\textbf{ASML}}{\textbf{ASL}}$
174	Cogent	6365	16644	2.61
3356	Level3	6074	26150	4.31
1299	TeliaNet	3776	11276	2.99
6939	Hurricane	3354	7844	2.34
2914	NTT	2517	7047	2.80
3257	GTT	2226	11107	4.99
7018	ATT	1922	5511	2.87
6461	Zayo	1521	3488	2.29
9002	RETN	1508	2274	1.51
4323	CenturyLink	1351	3845	2.85

IV. Experimental Results

In this section, we present our experimental results. For simplification, we use the terms ASL as AS level graphs and ASML as AS level multigraphs. To mimic the real-world Internet, we used real-world datasets presenting the current Internet topology. We used X-AS topology maps [1] to generate AS level graphs and multigraphs by using the following datasets.

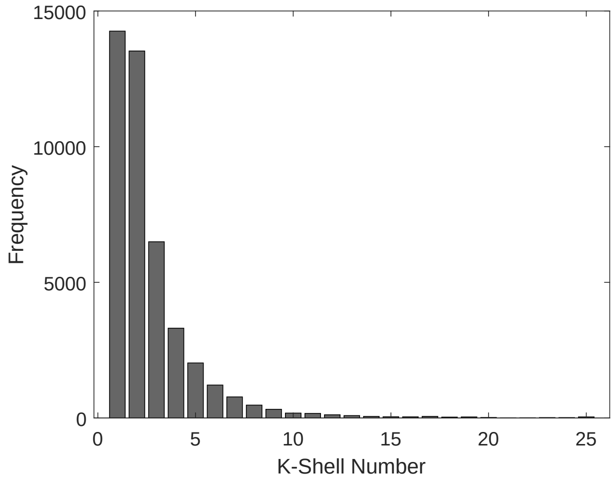
Traceroute: We used the CAIDA IPv4 Prefix-Probing Traceroute Dataset [23] consisting of more than 20 million path traces. The minimum and maximum Interface level hop lengths in our dataset are 1 and 31, respectively. The average hop length is 15.43.

IP2AS Mapping: RouteViews prefix to AS mapping dataset is obtained from CAIDA [24]. In order to generate an AS Level Internet topology, we mapped IP addresses reported in the traceroute dataset to their corresponding ASes. The dataset consists of 43,361 different ASes. The minimum and maximum AS level hop lengths in our dataset are 1 and 12, respectively. The average AS level hop length is 4.16.

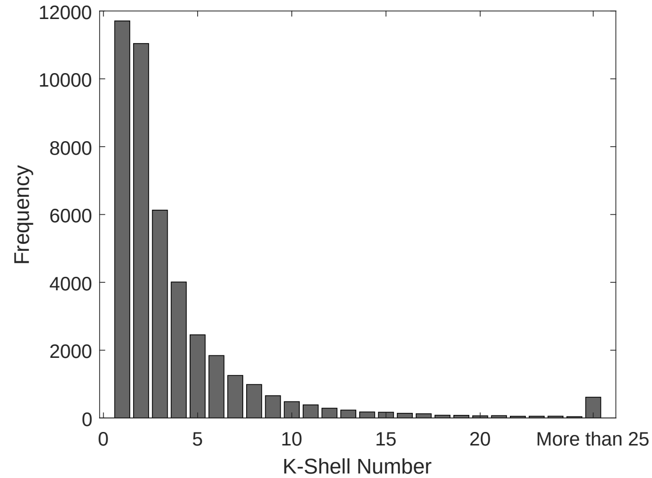
Geolocation Methods for X-AS Maps: Although DNS has limited support, it is still one of the most valuable information sources that directly comes from the ASes. ASes typically encode geographic information in their DNS naming conventions. We use UNDNS, which is a tool for extracting geolocation information from DNS names. It is developed as part of the RocketFuel project [18] and improved further by the iPlane project [19]. Additionally, we use 3 IP to Geolocation databases, including the commercial version of "DB-IP IP address to location" database [26], the free versions of "Maxmind GeoLite2 City" [27] and "IP2Location DB5 Lite" [25] databases. For more details, please refer to the X-AS paper [1].

A. Degree Distribution of AS Level Graphs and Multigraphs

We define the degree of an AS as the number of connections it has to other ASes. Figure 3 presents the degree distribution



(a) AS Level Graph (ASL)



(b) AS Level Multigraph (ASML)

Fig. 4: AS Level Graph and Multigraph K-Shell Decomposition of the Internet

TABLE III: Summary statistics for frequency

$Type$	Q_0	Q_1	Q_2	Q_3	Q_4	$Mean$	$StdDev$
ASL	1	1	2	3	25	2.69	2.44
ASML	1	1	2	4	1638	5.08	26.34

of AS level graphs and multigraphs. Additionally, Table I shows the summary statistics. It is clear that the majority of the ASes are virtually at the edge of the Internet without providing any internet access to other ASes. 95.1% of the ASes have a degree less than 10 in AS level whereas 91.2% in AS multilevel. Only 17 ASes have more than 1000 connections with other ASes in AS level, where the maximum degree is 6365. On the other hand, 35 ASes have more than 1000 connections with other ASes in AS multilevel, where the maximum degree is 26150. Table II shows the degrees for the top-10 ASes in the Internet. In AS Level, Cogent leads it with 6365 connections to other ASes. However, when we consider parallel and multiple connections between ASes, we observe that Level3 has almost two times more connections than Cogent. In the table, we defined a ratio, which is ASML divided by ASL, to show parallel connections rate. We observe that GTT has the largest rate of 4.99, whereas RETN has the lowest rate of 1.51.

B. K-Shell AS Frequency Analysis

In order to understand the structure of the Internet, we applied k-shell decomposition and analyzed the number of AS belongs to each shell. Figure 4a presents the AS frequency distribution of the Internet in AS level. We observe that 32.88% of the ASes are in 1-shell, and 31.19% are in 2-shell. In AS level, we observe the k_{max} is 25 where the 25-shell contains only 41 ASes, which corresponds to 0.09% of all ASes. Next, we analyze the AS level multigraphs in Figure 4b. We observe that only 27% of the ASes are in 1-shell, and

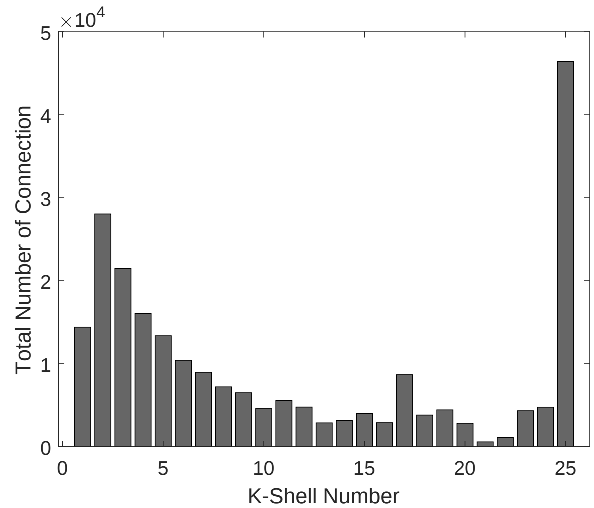


Fig. 5: K-Shell Connection analysis for AS Graphs

25.5% of the ASes are in 2-shell. 84 ASes have more than 25 shell number with a k_{max} 1638. Table III shows the summary statistics for AS level graphs and multigraphs. The mean of AS level graphs is 2.69, whereas AS multilevel is 5.08, almost two times more.

C. K-Shell Connection Analysis

In this part, we analyze the connections between ASes within each shell. The number of AS in each shell and the shell number has inverse relation where the number of AS is significantly low in higher-numbered shells. On the other hand, high-numbered shells are the leading ISPs that make the majority of the connections in the Internet. Figure 5 presents the AS level connection analysis of each shell. 27782 ASes belong to 1 and 2 shells, whereas only 41 AS is in the 25-shell. Even though the AS size is very large in the first two shells, the connection of 41 AS is more than the total of the first two

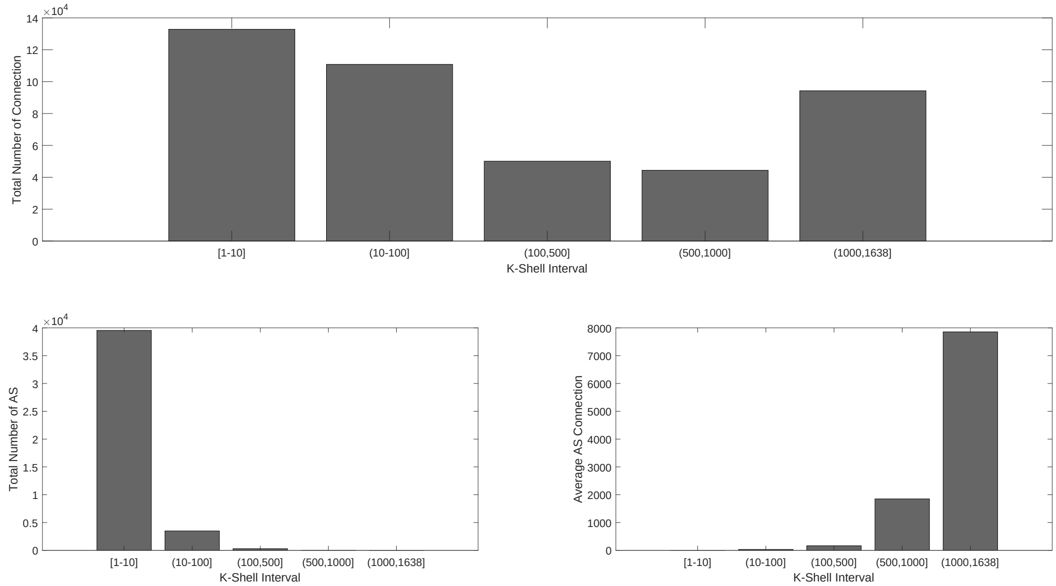


Fig. 6: K-Shell Connection analysis for AS Multigraphs

TABLE IV: Top-20 ASes detailed comparison (ASML Shell sorted)

ASN	Company	ASL Shell	ASML Shell	Cone Size	Cone Size Ranking	ASL Degree Ranking	ASML Degree Ranking	Customers	Providers	Peers
3356	Level3	25	1638	45771	1	2	1	5636	0	68
3549	Level3	25	1638	5280	13	9	7	1797	2	349
174	Cogent	25	1313	29747	3	1	2	6104	0	90
3257	GTT	25	1313	20668	5	5	4	1909	0	68
1299	TeliaNet	25	1063	34813	2	3	3	1986	0	51
6762	Telecom Italia	25	1063	17534	6	21	12	506	0	70
6939	Hurricane	25	1063	15095	7	4	5	1718	1	6752
701	MCI Comm	25	1063	3310	20	18	16	1381	0	34
209	CenturyLink	25	1063	2270	31	14	10	1595	0	57
3491	PCCW	25	948	9931	10	26	18	576	0	136
2914	NTT	25	818	20998	4	5	6	1643	0	63
6453	Tata	25	818	14593	8	16	14	659	0	55
6461	Zayo	25	818	12194	9	10	15	1801	0	281
3320	Deutsche Telekom	25	818	3327	19	25	19	581	0	68
7018	ATT	25	818	3236	21	6	8	2507	0	42
2828	MCI Comm	25	818	1614	38	15	9	582	0	42
20940	Akamai	25	818	12	1941	119	28	11	117	373
7922	Comcast	25	694	2483	27	17	17	106	4	63
1239	Sprint	25	692	2264	32	29	23	325	0	69
4134	ChinaNet	25	577	400	102	66	32	119	4	222

shells. Figure 6 presents the AS multilevel connection analysis of the Internet. We observe that 12 ISPs belong to shell number more than 1000 with 94265 connections and an average of 7855.42 connections. Additionally, 39538 ASes belongs to less than 10-shell with a total of 132849 connections and an average of 3.36 connections.

D. Analysis of Top-20 ISPs

To understand the main core of the Internet, we analyze the top-20 ASes with respect to their AS Multilevel shell number. Since all top-20 belong to the same 25-shell in AS level, we sorted the list based on ASML shell numbers. To provide more detail, we use CAIDA AS Ranking [28] to receive customer cone size, cone size ranking, number of customers, providers, and peers. Remember that, the customer-cone of an AS is the set of ASes consisting of the AS itself, its customer ASes, and the customer-cones of those customer ASes [14]. Finally, we included ASL degree ranking and ASML degree ranking that we discussed in subsection IV-A.

In our analysis, we observe a couple of interesting outcomes for some top-level ISPs. Our first observation is Level3 (ASN3356, ASN3349). Level3 has 5636 customers, 68 peers, and 0 providers and located in the top shell. Level3 makes several connections in different locations with many ISPs comparing to other ISPs. These additional connections also increase the redundancy, which improves the resilience and robustness of their network.

Next, Hurricane Electric (ASN6939) is defined as tier-2 AS in the traditional tier system because it has a c2p relation with Telia Company AB (ASN1299). However, when we analyze the Hurricane Electric network, we observe that they are located at 25-shell in AS level and 1063-shell in AS multilevel. They have 6752 peering relations with other ASes, which is the highest peering number in the Internet. Therefore, assessing the criticality of the Hurricane Electric network via the traditional tier system is neither adequate nor correct.

Our next observation is NTT America Inc (ASN2914). Their ASL degree ranking is 5 and ASML degree ranking is 6 among all ASes. However, when we analyze the ASML shell of NTT, we observe that 10 more ASes have a larger k value.

Finally, Akamai (ASN20940) has an ASL degree rank of 119 and an ASML degree rank of 28. Interestingly, it belongs to 25-shell in ASL and 818-shell in ASML. When we analyze their network in more detail, we observe that they make several connections in many ASes, which makes their network robust. Since most of the top ISPs give provider service to smaller ISPs, e.g., c2p relationship, we observe that Akamai has only 11 c2p relationships, whereas 117 providers and 373 peers. For a comparison purpose, AT&T (ASN7018) is in the same shell as Akamai, and ATT has 2507 customers, 0 provider, and 42 peers. The Akamai case shows us customer cone size is not always the best metric to classify ASes because some content delivery networks do not provide Internet connection to smaller ASes (fewer customers). On the other hand, they make several peering and provider relations to enable their services. Therefore, their customer cone size is small, but their criticality and connection sizes are larger than most top-level ISPs.

V. Conclusion

Categorization of ASes plays an essential role in understanding the structure and evolution of the Internet. However, the traditional categorization exhibits variation in different studies, contains ambiguity, involves subjectiveness, and sometimes does not match the reality. A better approach to classify ASes is defining the AS level topology maps as graphs and taking advantage of the graph properties through k-shell decomposition. In this work, we introduce k-shell decomposition on AS level multigraphs and comparisons with AS level graphs. Our analysis results help us to understand the structure of the Internet better.

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